Eur. Phys. J. A **5**, 117–121 (1999)

THE EUROPEAN PHYSICAL JOURNAL A

On the Coulomb corrections to the total cross section of the interaction of the $\pi^+\pi^-$ atom with ordinary atoms at high energy

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Received: 30 November 1998
Communicated by A. Schäfer

Abstract. The size of $\pi^+\pi^-$ atom in the low lying states is considerably smaller than the radius of atomic screening. Due to that we can neglect this screening calculating the contribution of multi-photon exchanges. We obtain the analytic formula for Coulomb corrections which works with a very good accuracy for the ground state of $\pi^+\pi^-$ atom.

The proposed accurate measurement of the $\pi^+\pi^-$ atom (dimesoatom) lifetime in the experiment DIRAC [1] would give important information about the low energy QCD dynamics. Dimesoatom can be detected in the experiment of a such type due to its (electromagnetic) interaction with the target matter (ordinary atoms). After this interaction dimesoatom can remain intact (elastic scattering), be excited in another quantum state (excitation) or to disintegrate (ionisation).

In this paper we will concentrate ourselves on the total cross section which is the sum of the total cross sections of elastic, excitation and ionisation. Total cross section, due to the optical theorem, is related to the imaginary part of the forward elastic scattering amplitude. At high energy it is given by the diagrams of Fig. 1 with the even number $N = 2n$ of photon exchanges in the t- channel.

In the Born approximation this process was considered in [2]. The contributions of the diagrams with multiphoton exchanges $n > 1$, Fig. 1(b), is important for an atom having large Z due to the strong Coulomb field of the nucleus. The expansion parameter $\nu = Z\alpha$ is not small in the case of the atom with large charge Z, therefore the contribution of all diagrams at Fig. 1 should be summed to achieve the exact result. Note that we consider here heavy atom as a projectile, or in other words, its quantum state does not change during the interaction. The grounds for this approximation will be discussed in the end of the paper.

When our paper was under preparation we learned about the paper [3] in which a similar ideas about the dominance of small transverse distances region to Coulomb corrections were proposed. Technically our method is different from that one used in [3], it looks for us more straightforward. Here we reproduce the result of [3], see our (12-15). Also we calculate the first correction to this result, see (29). It gives a possibility to estimate the accuracy of analytical approach and expand its applicability region from ground to low lying states of dimesoatom.

Let us divide the total cross section into two parts

$$\sigma = \sigma^{Born} + \sigma^{Coulomb} , \qquad (1)$$

where σ^{Born} is given by the contribution of the lowest order two photon exchange amplitude only, see Fig. 1(a), $\sigma^{Coulomb}$ describes the contribution of the diagrams Fig. 1(b) with $n > 1$ and their interference with the Born amplitude, Fig. 1(a).

It is known that the sum of type Fig. 1 diagrams in high energy kinematics is equivalent to the eikonal approximation. Recently this process was considered in the eikonal approach in [4]. In our note we would like to emphasize the physical picture underlying this process. Namely we discuss the hierarchy of transverse distances relevant for various contributions to the cross section: σ^{Born} and $\sigma^{Coulomb}$. In contrast to Born part, Coulomb correction receives the main contribution from the small distances where the electromagnetic field of atom is determined by the Coulomb field of nuclei. Therefore $\sigma^{Coulomb}$ can be calculated with high precision since the details of atomic screening in this case are of a small importance. Our consideration will be very similar to that in [5], where the closely related problem, lepton pair photoproduction in a strong Coulomb field, was considered. We derive the analytical formula which gives with good accuracy $\sigma^{Coulomb}$ for the ground state and describes qualitatively low lying dimesoatom states.

Electromagnetic field of atom consists of the field of nucleus and the field of electron shell. The nucleus field is

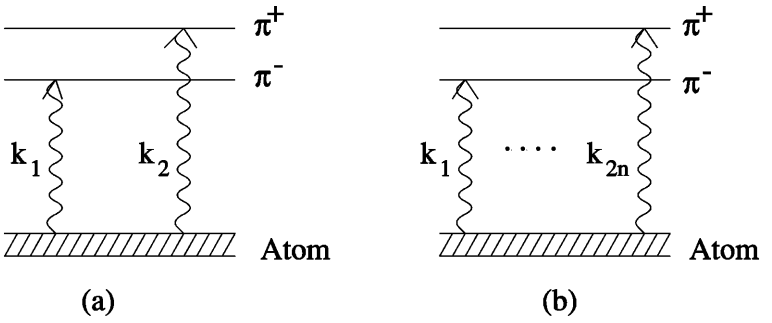

Fig. 1. Feynman diagrams describing the electromagnetic interaction of dimesoatom with the target atom: **a**) Born approximation, **b**) multiphoton contributions

screened by the electron shell at large distances

$$r \sim r_A = \frac{1}{m_e \alpha Z^{1/3}} \ . \tag{2}$$

At very small distances

$$r \leq r_N = \frac{1}{\Lambda} \ , \Lambda \approx 30 \text{ MeV} \ \ , \tag{3}$$

electromagnetic field depends on the distribution of electric charge inside nucleus. There is very broad region

$$r_N \ < \ r \ < \ r_A \tag{4}$$

where electromagnetic field of atom coincides with the Coulomb law $\frac{Z\alpha}{r}$.

The other important for our problem dimensional parameter is the distance between π^+ and π^- in dimesoatom, which is in a good approximation positronium–like weakly bound state,

$$r_{2\pi} \sim \frac{2}{m_\pi \alpha} \ . \tag{5}$$

It is important to note that this parameter lies in between of the parameters describing atomic and nuclear screening

$$r_N << r_{2\pi} << r_A \ . \tag{6}$$

Let us discuss now the relevant transverse momenta $< k_i >$ in the integrals describing the diagrams at Fig. 1, and therefore, the important for our process transverse distances $r \sim 1/ < k_i >$ between nucleus and high energy dimesoatom. These $< k_i >$ are different for the various contributions to the cross section. Calculating Born part σ^{Born} we meet the logarithmic–type integral collecting from the region

$$\frac{1}{r_A} \ \leq \ < k_{1,2} > \ \leq \ \frac{1}{r_{2\pi}} \ . \tag{7}$$

Therefore large distances, $r \sim r_A$, where the Coulomb field of nucleus is screened by the electron shell give sizeable contribution to the σ^{Born}. On the other hand $< k_i > \sim \frac{1}{r_{2\pi}}$ in the case of Coulomb contribution to the cross section and hence the typical transverse distances for $\sigma^{Coulomb}$ are of order of the dimesoatom size $r_{2\pi}$. Since there is a large gap between this size and r_A and r_N in calculation of $\sigma^{Coulomb}$ we can safely neglect the nuclear screening and with a good approximation the atomic screening. The accuracy of this approximation will be discussed later.

The total cross section for the interaction of dimesoatom with the target atom can be most easily obtained by exploiting the similarity between the cross section for this process (treating both π mesons as point like particles) and the cross section for the lepton pair production by an incident high-energy photon in a strong electromagnetic field of a nucleus. This lepton pair production occurs also via the multi-photon exchanges, analogous to those shown in Fig. 1 (with π^+, π^- mesons replaced by corresponding leptons). The total cross section for this process was calculated in [5] by direct summation of the perturbative series (see (48), (49), (52)). In order to make this above mentioned similarity between the both processes more explicit let us concentrate ourselves on e.g. the cross section for the lepton pair production by longitudinaly polarized incident photon σ^S. (The similar analysis can be of course performed in the case of transversely polarized photon.) This cross section σ^S is given in [5] by (49) and the first (52) (in which we do not subtract for this discussion the contribution of one photon exchange, i.e. we omit the last term on the r.h.s. of (52)). We can put this result in the form

$$\sigma^S = 2Re\alpha \int\limits_0^1 \frac{dx}{2\pi^2} \, d^2x_1 \, d^2x_2 \, 4Q^2x^2(1-x)^2 K_0^2(\mu|\boldsymbol{x_1} - \boldsymbol{x_2}|) \times \left[1 - \left(\frac{x_1^2}{x_2^2}\right)^{i\nu}\right] \tag{8}$$

where α is the fine structure constant, Q^2 is the virtuality of photon, $\boldsymbol{x_i}$, $i = 1, 2$, are the impact parameters of the leptons, the integration variable x is the fraction of longitudinal momentum of the pair carried by lepton ($1-x$ – by antilepton), $\mu^2 = m^2 + Q^2x(1-x)$, where m is the mass of the lepton, and K_0 is the modified Bessel function. Let us note, that since at high energy the diagrams with photon exchanges in the t- channel lead to the forward scattering amplitude proportional to the square of the scattering energy, the cross section itself (due to the optical theorem) does not depend on the scattering energy.

The physical meaning of the expression (8) can be made transparent if we introduce the square of the light-cone wave function of the fluctuations of a longitudinaly polarized virtual photon into lepton pair [6]

$$|\psi_S(x,|\boldsymbol{x_1}-\boldsymbol{x_2}|)|^2 = \frac{2\alpha}{\pi}\, 4Q^2x^2(1-x)^2 \times K_0^2(\mu|\boldsymbol{x_1}-\boldsymbol{x_2}|)\,, \qquad (9)$$

and the eikonal phases related to the Coulomb potential of the target atom

$$\chi(\boldsymbol{x}) = -\nu \ln \bar{\mu}^2|\boldsymbol{x}|^2 = \frac{1}{\pi}\int d^2\,k\; e^{i\boldsymbol{k}\boldsymbol{x}}\,\frac{\nu}{\boldsymbol{k}^2} = \int\limits_{-\infty}^{\infty} dz\,\frac{\nu}{\sqrt{\boldsymbol{x}^2+z^2}}\,, \qquad (10)$$

here $\bar{\mu}$ is an infrared cutoff. Using expressions (9) and (10) we can write the cross section (8) as

$$\sigma^S = 2Re\int\limits_0^1 \frac{dx\,d^2\,x_1\,d^2\,x_2}{4\pi}\,|\psi_S(x,|\boldsymbol{x_1}-\boldsymbol{x_2}|)|^2 \times\left[1-\exp\left(i\chi(\boldsymbol{x_2})-i\chi(\boldsymbol{x_1})\right)\right]\,, \qquad (11)$$

or after introducing the new variables $\boldsymbol{b} = \frac{\boldsymbol{x_1}+\boldsymbol{x_2}}{2}$ and $\boldsymbol{s} = \boldsymbol{x_1}-\boldsymbol{x_2}$, in the equivalent form

$$\sigma^S = 2Re\int\limits_0^1 \frac{dx\,d^2\,s}{4\pi}\,d^2\,b|\psi_S(x,|\boldsymbol{s}|)|^2 \times\left[1-\exp\left(i\chi(\boldsymbol{b}-\frac{1}{2}\boldsymbol{s})-i\chi(\boldsymbol{b}+\frac{1}{2}\boldsymbol{s})\right)\right]. \qquad (12)$$

We see therefore that the cross section (12) is given as a convolution of the square of the wave function for lepton pair fluctuation of the photon with the eikonal amplitude describing the multi-photon exchanges between the leptonic lines and the target atom. This structure of the cross section is well known from the Glauber theory which we recover in this case. On the basis of this observation we can write the cross section for the interaction of dimesoatom with the target atom in a similar form

$$\sigma_{nlm} = 2\,\mathrm{Re}\int d^2b\,d^3r|\psi_{nlm}(\boldsymbol{r})|^2 \times\left[1-\exp\left(i\chi(\boldsymbol{b}-\boldsymbol{s}/2)-i\chi(\boldsymbol{b}+\boldsymbol{s}/2)\right)\right]. \qquad (13)$$

Here $\boldsymbol{s}=\boldsymbol{r}_\perp$ is the projection of the vector $\boldsymbol{r}$ on the plane perpendicular to the collision axis, the impact parameter of dimesoatom is $\boldsymbol{b}$, $\psi_{nlm}(\boldsymbol{r})$ is the wave function of $\pi^+\pi^-$ atom in the state with principal, orbital and magnetic quantum numbers n, l and m respectively. The phase shift $\chi(\boldsymbol{b})$ is expressed via potential of the target atom (compare (10)):

$$\chi(\boldsymbol{b}) = \int\limits_{-\infty}^{\infty} U(\sqrt{b^2+z^2}\,)\,dz\,. \qquad (14)$$

This equation was derived in [7] and subsequently used in the analysis of [4]. It serves also as the starting point for our forthcoming analysis.

As we have seen already in (10), the difference between the phase shifts of π^+ and π^- can be easily calculated in the case of Coulomb potential. Therefore

$$\sigma_{nlm} = 2\,\mathrm{Re}\int d^2b\,d^3r|\psi_{nlm}(\boldsymbol{r})|^2 \times\left[1-\left(\frac{(\boldsymbol{b}-\boldsymbol{s}/2)^2}{(\boldsymbol{b}+\boldsymbol{s}/2)^2}\right)^{i\nu}\right]. \qquad (15)$$

This integral becomes convergent after the subtraction of the Born contribution which is divergent in the case of unscreened Coulomb potential.

$$\sigma_{nlm}^{Coulomb} = 2\,\mathrm{Re}\int d^2b\,d^3r|\psi_{nlm}(\boldsymbol{r})|^2 \times\left[1-\left(\frac{(\boldsymbol{b}-\boldsymbol{s}/2)^2}{(\boldsymbol{b}+\boldsymbol{s}/2)^2}\right)^{i\nu}-\frac{\nu^2}{2}\ln^2\left(\frac{(\boldsymbol{b}-\boldsymbol{s}/2)^2}{(\boldsymbol{b}+\boldsymbol{s}/2)^2}\right)\right]. \qquad (16)$$

After the substitution $\boldsymbol{b}\to s(\boldsymbol{R}-\boldsymbol{n})$, where $\boldsymbol{n}=\frac{\boldsymbol{s}}{s}$, the integral in (16) factorizes, see also [5].

$$\sigma_{nlm}^{Coulomb} = \langle s^2\rangle I_\nu\,, \qquad (17)$$

where

$$I_\nu = \int d^2\boldsymbol{R}\left\{2-\left(\frac{R^2}{(\boldsymbol{R}-\boldsymbol{n})^2}\right)^{i\nu} -\left(\frac{R^2}{(\boldsymbol{R}-\boldsymbol{n})^2}\right)^{-i\nu}-\nu^2\ln^2\left(\frac{R^2}{(\boldsymbol{R}-\boldsymbol{n})^2}\right)\right\} \qquad (18)$$

and

$$\langle s^2\rangle = \int d^3r r^2\sin^2\Theta|\psi_{nlm}(\boldsymbol{r})|^2 = \langle r^2\rangle_{(n,l)}\langle\sin^2\Theta\rangle_{(l,m)} \qquad (19)$$

The integral I_ν was calculated in [5]

$$I_\nu = -4\pi\nu^2 f(\nu),$$
$$f(\nu) = \frac{1}{2}[\Psi(1-i\nu)+\Psi(1+i\nu)-2\Psi(1)], \qquad (20)$$

where $\Psi(z) = d(\ln\Gamma(z))/dz$. Note that the dependence of the $\sigma^{Coulomb}$ on Z factorizes from the variables describing the state of $\pi^+\pi^-$ atom. It is given by the universal function $f(\nu)$.

$\langle r^2\rangle_{(n,l)}$ for the positronium–like states is given by [8]

$$\langle r^2\rangle_{(n,l)} = \left(\frac{2}{m_\pi\alpha}\right)^2\frac{n^2}{2}[5n^2+1-3l(l+1)] \qquad (21)$$

We will consider for simplicity the cross section averaged over the magnet quantum number:

$$\sigma_{nl} = \frac{1}{2l+1}\sum_m \sigma_{nlm}\,. \qquad (22)$$

In this case

$$\langle \sin^2\Theta\rangle = 2/3\ . \tag{23}$$

Taking into account all factors we find the result

$$\sigma_{nl} = \sigma_{nl}^{Born} + \sigma_{nl}^{Coulomb}$$
$$\sigma_{nl}^{Coulomb} = -\frac{16\pi\nu^2 f(\nu)}{m_\pi^2\alpha^2}\frac{n^2}{3}[5n^2+1-3l(l+1)] \tag{24}$$

Note that $\sigma^{Coulomb}$ is proportional to $r_{2\pi}^2$, the mean square of the distance between π^+ and π^- in the dimesoatom. $r_{2\pi}^2$ grows rapidly, $\sim n^4$, with increasing n for weakly bounded dimesoatom. At $n\sim 4$ the distance between $\pi^+\pi^-$ becomes of the order of the radius of atomic screening r_A for atom with large Z. Therefore our approach based on the large difference between $r_{2\pi}$ and r_A can not be applied to the highly excited states of dimesoatom.

The first correction related to appearance of the atomic screening can also be calculated analytically. In order to evaluate it let us replace the denominator of the photon propagator by

$$\frac{1}{\mathbf{k}^2}\to\frac{1}{\mathbf{k}^2+\mu^2}\ , \tag{25}$$

where μ is the inverse of the radius of atomic screening r_A. This replacement leads to the analog of the formula (16) having now the form

$$\sigma_{nlm}^{Coul.+Atom.Scr.} = 2\,\mathrm{Re}\int d^2b\,d^3r|\psi_{nlm}(\boldsymbol{r})|^2 \times\Big[1-e^{2i\nu[K_0(\mu|\boldsymbol{b}-\frac{\boldsymbol{s}}{2}|)-K_0(\mu|\boldsymbol{b}+\frac{\boldsymbol{s}}{2}|)]} - 2\nu^2[K_0(\mu|\boldsymbol{b}-\frac{\boldsymbol{s}}{2}|)-K_0(\mu|\boldsymbol{b}+\frac{\boldsymbol{s}}{2}|)]^2\Big], \tag{26}$$

where $K_0(z)$ is the modified Bessel function.

The first correction due to the atomic screening can be obtained from (26) by taking into account that for small values of μ

$$K_0(\mu|\boldsymbol{b}-\frac{\boldsymbol{s}}{2}|)-K_0(\mu|\boldsymbol{b}+\frac{\boldsymbol{s}}{2}|) = \ln\frac{|\boldsymbol{b}+\frac{\boldsymbol{s}}{2}|}{|\boldsymbol{b}-\frac{\boldsymbol{s}}{2}|} + \frac{\mu^2}{4}\Big[(|\boldsymbol{b}-\frac{\boldsymbol{s}}{2}|^2-|\boldsymbol{b}+\frac{\boldsymbol{s}}{2}|^2)\Psi(2) + |\boldsymbol{b}+\frac{\boldsymbol{s}}{2}|^2\ln\frac{\mu|\boldsymbol{b}+\frac{\boldsymbol{s}}{2}|}{2} - |\boldsymbol{b}-\frac{\boldsymbol{s}}{2}|^2\ln\frac{\mu|\boldsymbol{b}-\frac{\boldsymbol{s}}{2}|}{2}\Big], \tag{27}$$

and keeping terms proportional to μ^2. In this way we obtain

$$\Delta\sigma_{nlm}^{Atom.Scr.} = -2\mu^2\,\mathrm{Re}\int d^2b\,d^3r|\psi_{nlm}(\boldsymbol{r})|^2 \times\Big[\frac{i\nu}{2}e^{2i\nu\ln\frac{|\boldsymbol{b}+\frac{\boldsymbol{s}}{2}|}{|\boldsymbol{b}-\frac{\boldsymbol{s}}{2}|}}+\frac{\nu^2}{2}\ln\frac{|\boldsymbol{b}+\frac{\boldsymbol{s}}{2}|^2}{|\boldsymbol{b}-\frac{\boldsymbol{s}}{2}|^2}\Big]\cdot \times\Big[(|\boldsymbol{b}-\frac{\boldsymbol{s}}{2}|^2-|\boldsymbol{b}+\frac{\boldsymbol{s}}{2}|^2)\Psi(2)+|\boldsymbol{b}+\frac{\boldsymbol{s}}{2}|^2\ln\frac{\mu|\boldsymbol{b}+\frac{\boldsymbol{s}}{2}|}{2} - |\boldsymbol{b}-\frac{\boldsymbol{s}}{2}|^2\ln\frac{\mu|\boldsymbol{b}-\frac{\boldsymbol{s}}{2}|}{2}\Big]\ . \tag{28}$$

The dominant contribution to this expression comes from the region of values of $\boldsymbol{b}$ being much lager than the size of the dimesoatom described by $\boldsymbol{s}=\boldsymbol{r}_\perp$. In this limit we obtain

$$\Delta\sigma_{nlm}^{Atom.Scr.} \approx \frac{8}{3}\mu^2\nu^4\int d^3r\int_r^{r_A} d^2b|\psi_{nlm}(\boldsymbol{r})|^2\,\frac{(\boldsymbol{bs})^4}{(\boldsymbol{b}^2)^3} \times\Big[\Psi(2)-\frac{1}{2}\ln\frac{\mu^2\boldsymbol{b}^2}{4}-\frac{1}{2}\Big]\ , \tag{29}$$

and consequently with the logarithmic accuracy we can write

$$\Delta\sigma_{nlm}^{Atom.Scr.}\approx\pi\mu^2\nu^4\int d^3r\,|\psi_{nlm}(\boldsymbol{r})|^2\,r^4\sin^4\Theta\ln^2(\mu r). \tag{30}$$

This result can be generalized to the case in which there appear a sum of several potentials, i.e. when instead of (25) we perform the substitution

$$\frac{1}{\mathbf{k}^2}\to\sum_i^N\frac{c_i}{\mathbf{k}^2+\mu_i^2}\ ,\quad \sum_i^N c_i=1\ . \tag{31}$$

This generalization includes, in particular, the Moliére [9] parametrization of the Thomas-Fermi potential. In this case the (30) generalizes to the formula

$$\Delta\sigma_{nlm}^{Atom.Scr.} \approx \pi\nu^4\int d^3r\,|\psi_{nlm}(\boldsymbol{r})|^2\,r^4\sin^4\Theta \times\sum_i^N\frac{1}{4}c_i\mu_i^2\ln^2(\mu_i^2r^2)\ . \tag{32}$$

Since the derivation of (30) was performed with the logarithmic accuracy we are free to put in the argument of logarithmic function in (32) some average value $\bar\mu^2$ of the square of masses μ_i. We choose as $\bar\mu^2$ the quantity

$$\bar\mu^2=\sum_i^N c_i\mu_i^2\ . \tag{33}$$

In this way we arrive to the final form of correction related to the atomic screening averaged over the magnet quantum number m (see (22))

$$\Delta\sigma_{nl}^{Atom.Scr.} \approx \frac{1}{4}\pi\nu^4\left(\sum_i^N c_i\mu_i^2\right)\,\frac{1}{2l+1}\sum_{m=-l}^{l} \times\int d^3r\,|\psi_{nlm}(\boldsymbol{r})|^2\,r^4\ln^2(\bar\mu^2r^2)\sin^4\Theta \approx\frac{2}{15}\pi\nu^4\left(\sum_i^N c_i\mu_i^2\right)\langle r^4\rangle_{n,l}\ln^2(\bar\mu^2\langle r^2\rangle_{n,l}\Big) \tag{34}$$

The cross section under consideration is given by the sum of terms

$$\sigma_{nl}=\sigma_{nl}^{Born}+\sigma_{nl}^{Coulomb}+\Delta\sigma_{nl}^{Atom.Scr.}\ , \tag{35}$$

Table 1. Predictions for $(\sigma_{nl}^{Born} - \sigma_{nl})/\sigma_{nl}^{Born}$ based on the analytic results in case of Tantalum and their comparison with the numerical results of [4]

	(24)	(35)	Fig. 2 of [4]
$n = 1, l = 0$	0.0851	0.08212	0.082
$n = 2, l = 0$	0.1321	0.1265	0.109
$n = 3, l = 0$	0.1962	0.1961	0.128
$n = 4, l = 0$	0.2846	0.1812	0.138

where $\sigma_{nl}^{Coulomb}$ is given by (24) and (20), and $\Delta\sigma_{nl}^{Atom.Scr.}$ is given by (34).

Let us now present for the case of Tantalum ($Z = 73$) our predictions for $(\sigma_{nl}^{Born} - \sigma_{nl})/\sigma_{nl}^{Born}$ based on the analytic results obtained above. The values of σ_{nl}^{Born} for different values of n and $l = 0$ we take from the first column of the Table in [2]. The values of constants c_i and masses μ_i were derived by Moliére in [9] (see (7,1a) there) and are equal

$$\begin{aligned} &c_1 = 0.35 \quad c_2 = 0.55 \quad c_3 = 0.1 \\ &\mu_1 = 0.3\mu_0 \quad \mu_2 = 1.2\mu_0 \quad \mu_3 = 6\mu_0 \\ &\text{where} \quad \mu_0 = \frac{m_e \alpha Z^{1/3}}{0.885} \ . \end{aligned} \tag{36}$$

Our predictions based on (24) and (35) are presented in the first two columns of Table 1.

Our analytic results we can compare with the results of numerical calculations presented on Fig. 2 of [4], and which are shown in third column of Table 1. These numerical results are based on (13) and use in a course of the calculations the explicit form of the pionium wave function (written in terms of the spherical harmonics and Laguerre polynomials) and the Moliére parametrization of the Thomas-Fermi potential given by (31) and (36).

We see that for the ground state of dimesoatom the first atomic screening correction reduces practically to zero the difference between result of our analytic calculations (24) and the result of numerical calculation of [4]. For the exited states this correction leads to decreasing of this difference but the precision is low. Consequently, for those states the analytic approach based on (24) or (35) works only qualitatively.

We want still to add three additional remarks. First, let us note that the nuclear screening can be really safely neglected since the corresponding relative accuracy is very high

$$\sim \left(\frac{r_N}{r_{2\pi}}\right)^2 < 10^{-4} \ . \tag{37}$$

Secondly, due to the fact that the values of muon and pion masses are close to each other, our qualitative consideration and its consequence given by (24),(34) are directly applicable to the process of the breakup of $\mu^+\mu^-$ atom.

Finally, we consider here an atom as a permanent source of electromagnetic field or as a projectile. The grounds of this approximation can be understood if we compare the estimates for the time of the interaction of pionium with an atom with the internal atomic time. The interaction time $\sim 1/(m_\pi \alpha)$ is much shorter than the internal atomic time $\sim 1/(m_e \alpha^2 Z^{4/3})$. An atom will remain in the initial state during the time of interaction, the change of its quantum state and therefore its electromagnetic field will occur only after a time of order of internal atomic time, therefore we can consider an atom as a permanent source of electromagnetic field.

We would like to acknowledge V.G. Serbo for useful discussions.

D.Yu.I. would like to acknowledge the hospitality at Soltan Institute for Nuclear Studies in Warsaw where this work started. His work is also supported by Russian Foundation for Basic Research, grant N 96-02-19114.

L.Sz. would like to acknowledge the support from German-Polish agreement on scientific and technological cooperation N-115-95.

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